# Tutorial 1: Modelling and basics Artificial Intelligence

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# Modelling

#### **Exercise 1**

Two players have 2 or more lots of matches. At each turn, the next player may remove a number of matches of a lot (depending on the selected rule). The player who removes the last match loses. Model this game with a graph in the case where one has from two piles each containing three matches, and where a player can remove one or two matches each. What move the first player must play to win the game?

### **Exercise 2**

The following graph represents the corridors of a museum. A guard placed in a corridor can monitor two junctions placed at its ends. How many guards are needed (and how to place them) so that all intersections are monitored?

If we now place guards at intersections, assuming that a guard can monitor all corridors leading to this crossroads, how many guards are needed?



### **Exercise 3**

Is it possible to chart 5 segments on a sheet of paper where each segment cuts exactly 3 others? Prove that in a non-oriented graph G = (V, E), the sum of all degrees is  $2^*|E|$ .

#### **Exercise 4**

Skynet is a network with 15 vertices. You can go from every vertex to at least seven other vertices by link. Can we go by link from a vertex X to each of the others?

#### **Exercise 5**

A network is composed of seven plants. A city is linked to exactly two plants. Two plants share exactly one city. How many cities are served by the network? Prove that a complete graph with n vertices has  $\frac{n(n-1)}{2}$  links.

#### **Exercise 6**

Considering the following mathematical system:

$$\begin{cases} X_1 + X_2 \leq 1 \\ X_1 + X_3 + X_4 \leq 1 \\ X_4 + X_5 \leq 1 \end{cases}$$

Where  $X_i$  is 0 or 1. Determine, with a graph problem, a solution that maximizes the objective function:  $Z = X_1 + X_2 + X_3 + X_4 + X_5$ .

#### **Exercise 7**

Considering a dominoes game using the numbers 0, 1, 2, 3, 4, as on each domino include two distinct digits, such as 1 and 3, the following problem is proposed: Is it possible to align all the dominoes so that when two pawns "touch" the numbers "in contact" are identical?

### **Exercise 8**

A group of 9 students met every day at a round table. How many days can they come together if we want anyone not having twice the same neighbor?

# Tree

## **Exercise 9**

Let G = (S, A) be a non-oriented graph with n vertices. Prove that all the following properties are equivalent:

- G is a tree
- *G* is connected and if we remove an edge, *G* is no longer connected
- *G* is connected with n 1 edge
- *G* has no cycle until we add an edge
- G has no cycle with n-1 edge
- Only one path between any couple of vertices

## **Exercise 10**

To increase efficiency of local plants, a company needs to link them in a connected network. The cost of the line between a couple of plants is shown in this adjacency matrix (lines are non-directed):

-	4	8	-	-	-	-	-	-
	-	11	-	8	-	-	-	-
		-	7	-	-	-	-	1
			-	2	-	-	-	6
				-	7	-	4	-
					-	9	14	-
						-	10	-
							-	2
								-

Construct an associated graph. Propose an algorithm to build a tree. How to solve the problem at the minimum cost? Give a solution. Found a solution with the maximum cost.

### **Exercise 11**

We want to add a battery in a network. The following graph shows cost to send energy between two substations in the network, and the amount of energy sent by a substation. You have to place the battery at a substation while minimizing total cost.



# **Graph coloring**

# **Exercise 12**

A server can route a maximum of x packages at the same time. Seven substations are linked to a server, a substation cannot send packages if some substations already use the link. The next table presents each substation of the ability to send a package in function of the other ones. For example, a package from A can be sent if there is already a package from D but cannot be sent when B sent a package.

Substation	А	В	С	D	E	F	G
Is allowed with	D,E	D,E,F,G	E,G	A,B,E	A,B,C,D,F,G	B,E,G	B,C,E,F

Represent the links in a graph. How many packages the server has to manage at the same time (maximum value)?

### **Exercise 13**

A school must pass written tests to four students: Pierre, Jean, Guillermo and Ibrahim. Seven disciplines are involved: mathematics, physics, biology, French, English, Spanish and history. Pierre must pass the mathematics, physics and English, Jean mathematics, biology and French, Guillermo mathematics, English and Spanish and Ibrahim physics, French and history. What is the minimum number of time slots to be expected for no student has to pass two tests simultaneously? What is the chromatic number of a complete graph? How to bound chromatic number in a graph?