

Practice 1.2: Modelling and basics

Artificial Intelligence

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Prim's Algorithm

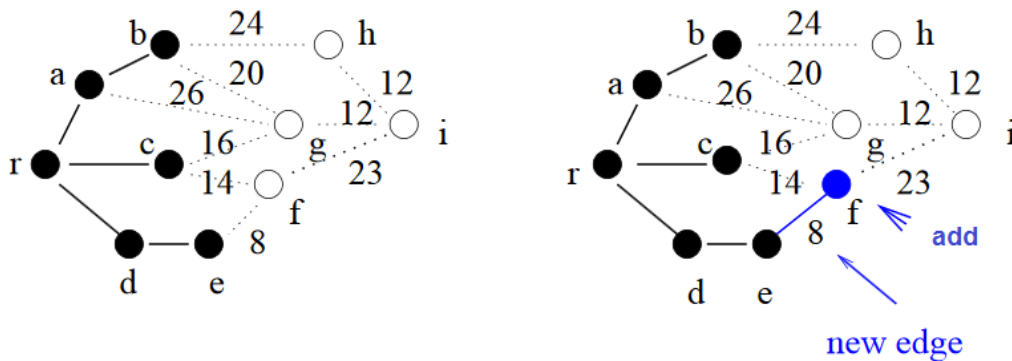
We consider a weighted connected graph G With n vertices. The Prim's algorithm finds a minimum spanning tree of G .

Step 0: Choose any element r ; set $S = \{r\}$ and $A = \emptyset$. (Take r as the root of our spanning tree.)

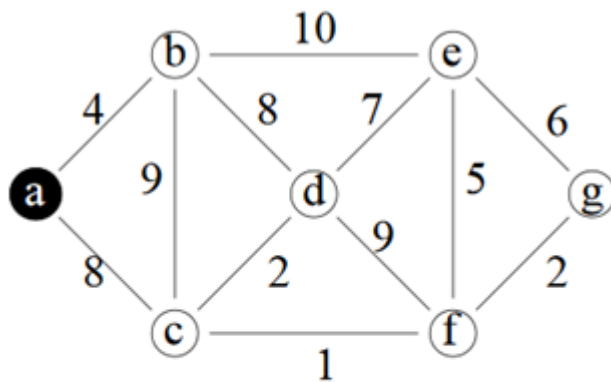
Step 1: Find a lightest edge such that one endpoint is in S and the other is in $V \setminus S$. Add this edge to A and its (other) endpoint to S .

Step 2: If $V \setminus S = \emptyset$, then stop & output (minimum) spanning tree (S, A) . Otherwise go to Step 1.

The idea: expand the current tree by adding the lightest edge leaving it and its endpoint.



Example

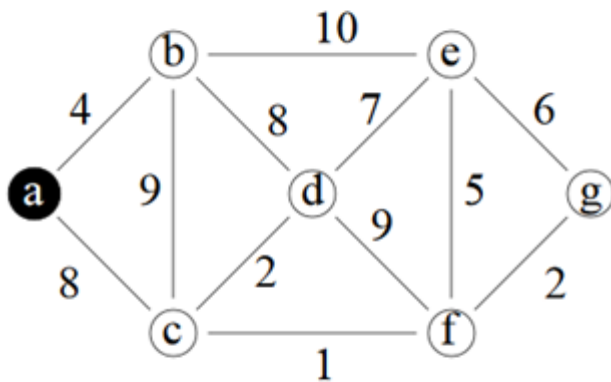


Step 0

$S = \{a\}$

$V \setminus S = \{b, c, d, e, f, g\}$

lightest edge = $\{a, b\}$



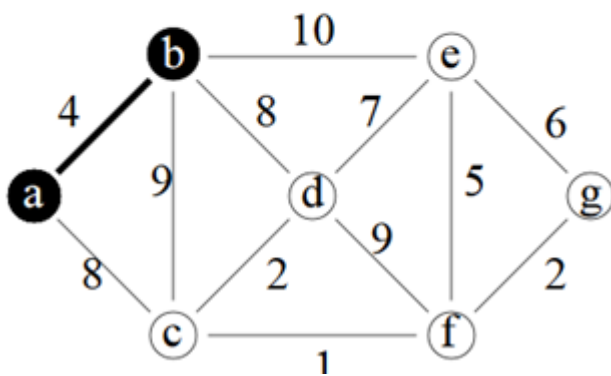
Step 1

$S = \{a\}$

$V \setminus S = \{b, c, d, e, f, g\}$

$A = \{\}$

lightest edge = $\{a, b\}$



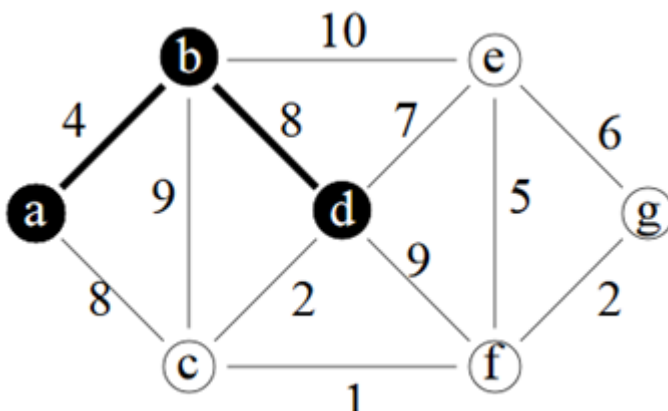
Step 1

$S = \{a, b\}$

$V \setminus S = \{c, d, e, f, g\}$

$A = \{\{a, b\}\}$

lightest edge = $\{b, d\}, \{a, c\}$



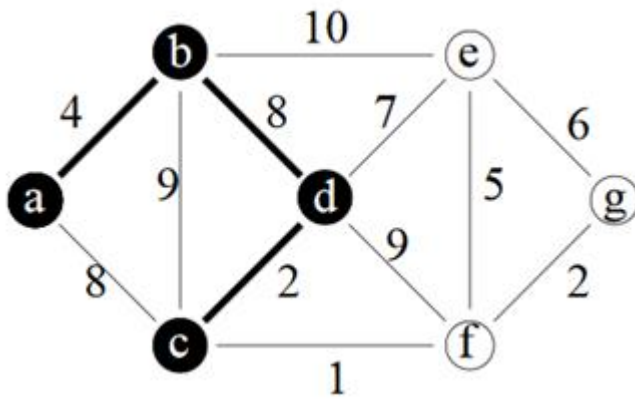
Step 1

$S = \{a, b, d\}$

$V \setminus S = \{c, e, f, g\}$

$A = \{\{a, b\}, \{b, d\}\}$

lightest edge = $\{d, c\}$



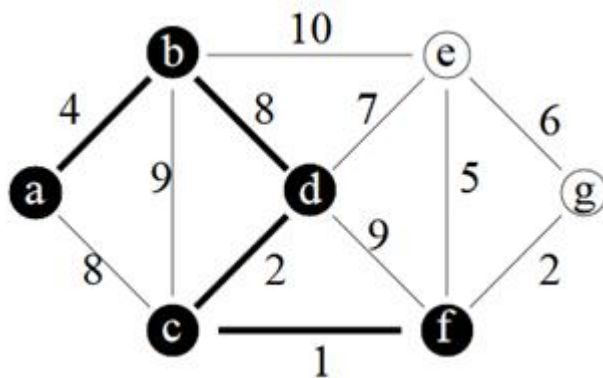
Step 1

$S = \{a, b, c, d\}$

$V \setminus S = \{e, f, g\}$

$A = \{\{a, b\}, \{b, d\}, \{c, d\}\}$

lightest edge = $\{c, f\}$



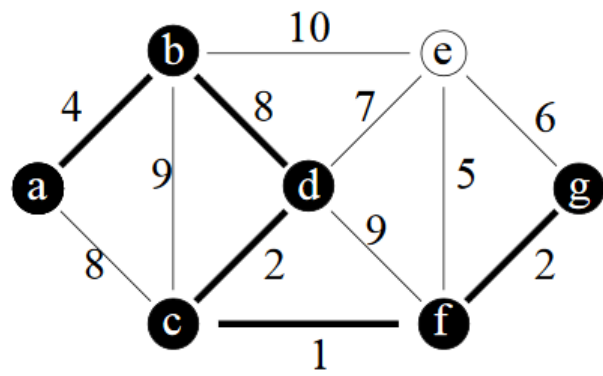
Step 1

$S = \{a, b, c, d, f\}$

$V \setminus S = \{e, g\}$

$A = \{\{a, b\}, \{b, d\}, \{c, d\}, \{c, f\}\}$

lightest edge = $\{f, g\}$



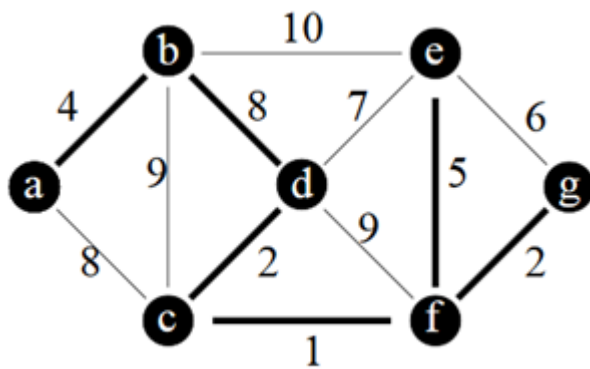
Step 1

$S = \{a, b, c, d, f, g\}$

$V \setminus S = \{e\}$

$A = \{\{a, b\}, \{b, d\}, \{c, d\}, \{c, f\}, \{f, g\}\}$

lightest edge = $\{f, e\}$



Step 1

$S = \{a, b, c, d, e, f, g\}$

$V \setminus S = \{\}$

$A = \{\{a, b\}, \{b, d\}, \{c, d\}, \{c, f\}, \{f, g\}, \{f, e\}\}$

MST completed

Exercise

Find a MST on these graphs:

