Practice 1.4: Modelling and basics

Artificial Intelligence

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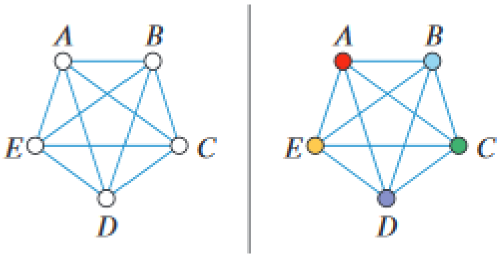
Exercise 1

Explain why $X(K_n) = n$ where K_n denotes the complete graph on n vertices.

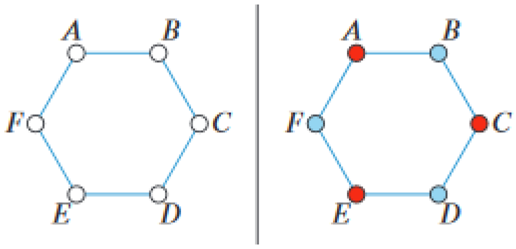
The graph K_5 , for example, is the complete graph on 5 vertices. In this graph, every vertex is adjacent to every other vertex, so no two vertices can have the same color. The only possible way to color is to use a different color for each vertex. Thus, we can conclude that $X(K_5) = 5$.

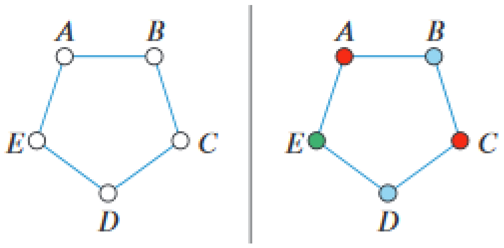


Exercise 2

Explain why $X(G) = 2$ when G is a path, and $2 \leq X(G) \leq 3$ when G is a circuit.

When G is a path, it's also a tree. Add the color 1 to odd distance node, and color 2 to the others. The chromatic number is equal to 2. When G is a circuit, we can conclude the chromatic number from a path deducted from the circuit (remove one edge from a circuit to form a path). Then, this path has $X(G) = 2$. When you add the edge, if the vertices are odd, the two vertices incident to the edge have the same color, we can't choose color 2 because of the adjacent vertex, thus we have to add a third color.

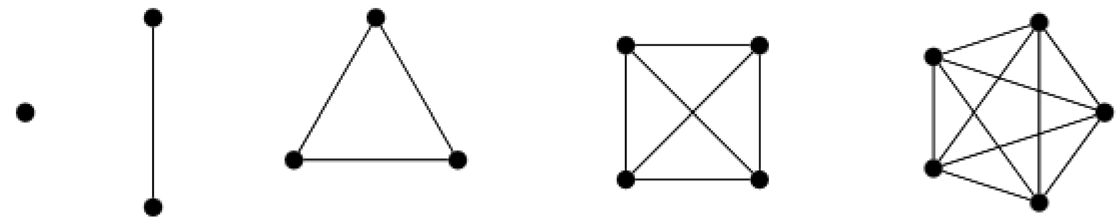




Exercise 3

Find the chromatic number of the following graphs.

Complete Graphs

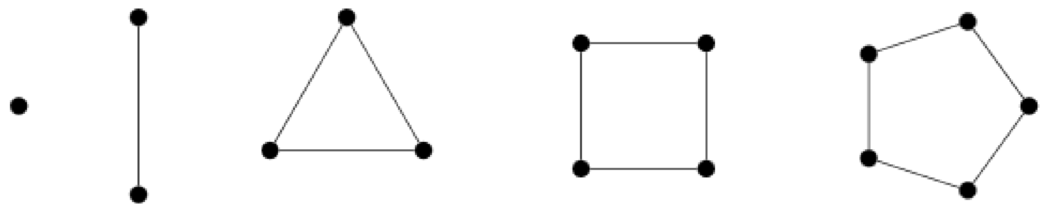


K_1 K_2 K_3 K_4 K_5

Paths

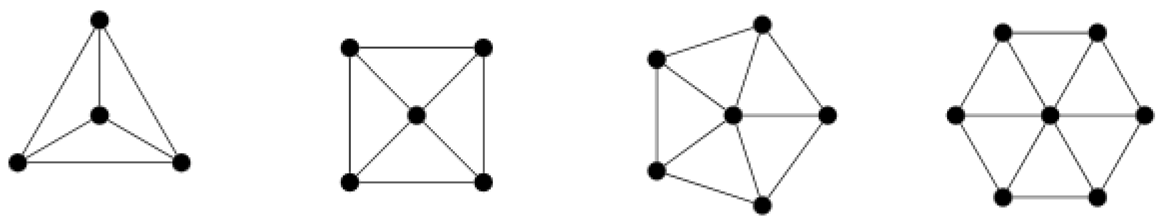
P_1 P_2 P_3 P_4

Cycles



C_1 C_2 C_3 C_4 C_5

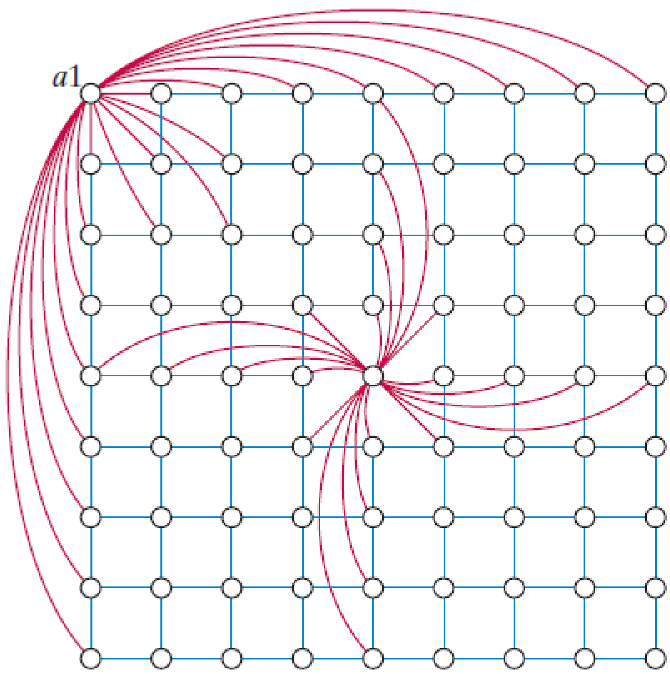
Wheels



See more… SUDOKU

	1	2	3	4	5	6	7	8	9
a			4	8					
b		9		4	6			7	
c		5					6	1	4
d	2	1		6			5		
e	5	8		7		9		4	1
f			7			8		6	9
g	3	4	5					9	
h		6			3	7		2	
i						4	1		

To see the connection between Sudoku and graph coloring, we will first describe the Sudoku graph, which for convenience we will refer to as S. The graph S has 81 vertices, with each vertex representing a cell. When two cells cannot have the same number (either because they are in the same row, in the same column, or in the same box) we put an edge connecting the corresponding vertices of the Sudoku graph S. For example, since cells a3 and a7 are in the same row, there is an edge joining their corresponding vertices; there is also an edge connecting a1 and b3 (they are in the same box), and so on. When everything is said and done, each vertex of the Sudoku graph has degree 20, and the graph has a total of 810 edges. S is too large to draw, but we can get a sense of the structure of S by looking at a partial drawing. The drawing shows all 81 vertices of S, but only two (a1 and e5) have their full set of incident edges showing.



The second step in converting a Sudoku puzzle into a graph coloring problem is to assign colors to the numbers 1 through 9. This assignment is arbitrary, and is not a priority ordering of the colors as in the greedy algorithm, it's just a simple correspondence between numbers and colors.

Once we have the Sudoku graph and an assignment of colors to the numbers 1 through 9, any Sudoku puzzle can be described by a Sudoku graph where some of the vertices are already colored (the ones corresponding to the givens).To solve the Sudoku puzzle all we have to do is color the rest of the vertices using the nine colors.