Practice 4.2: Flow

Artificial Intelligence

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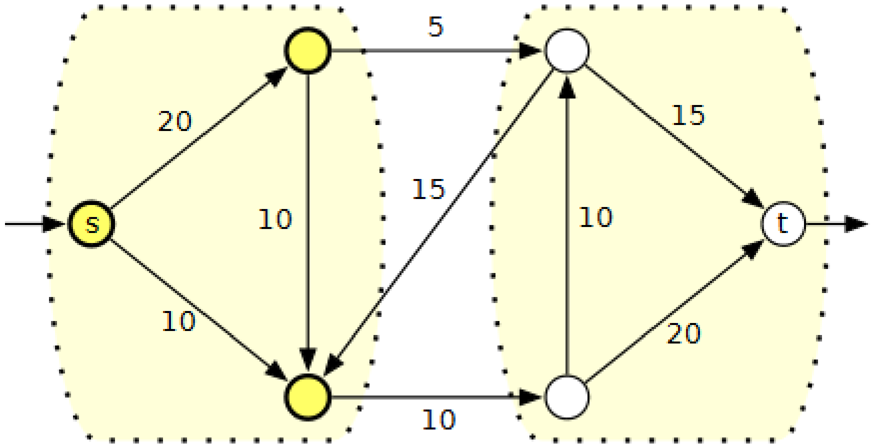
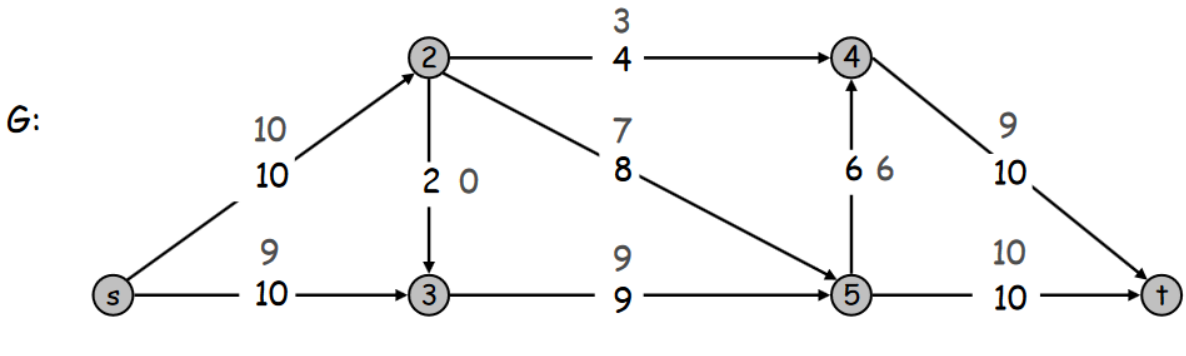
Ford Fulkerson's algorithm: Min-Cut analysis

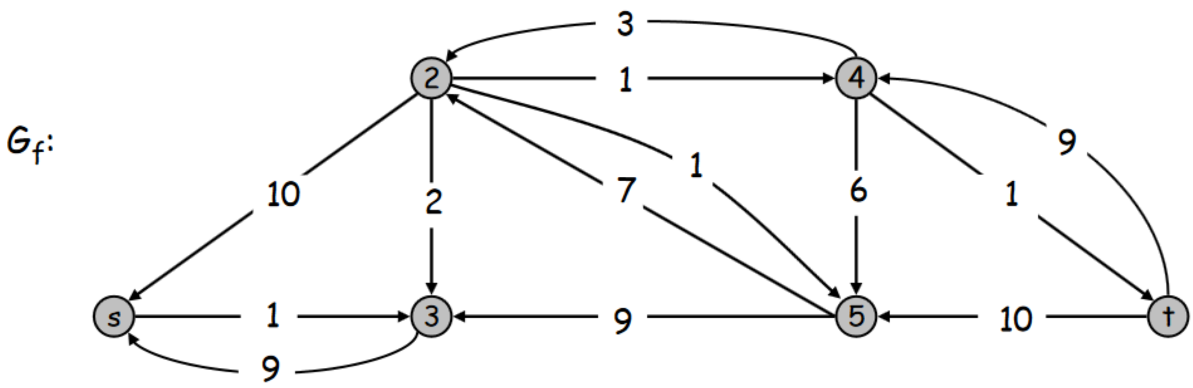
Intuitively, the minimum cut is the cheapest way to disrupt all flow from S to t. Indeed, it is not hard to show that the value of any feasible (s,t)-flow is at most the capacity of any (s,t)-cut



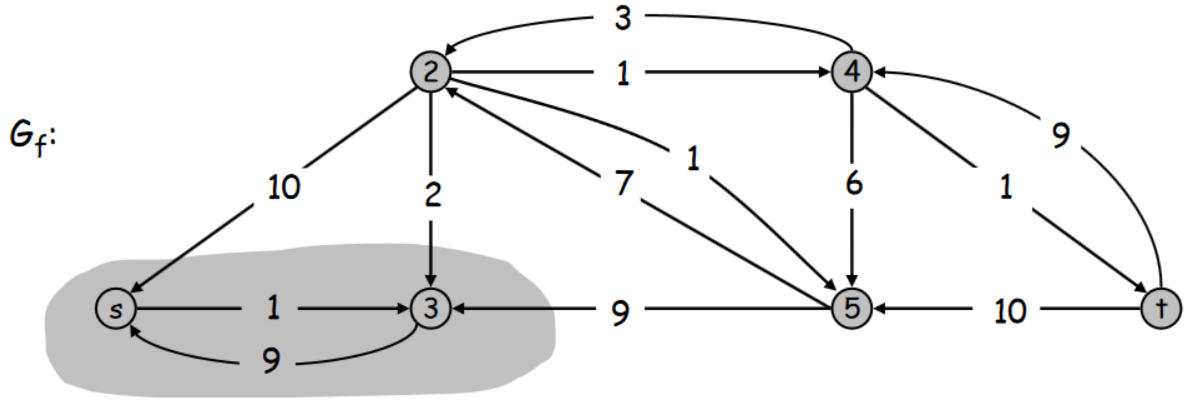
By definition, $|f| = \sum_w f(s \to w) - \sum_u f(u \to s)$. We increase the value of the flow if and only if f saturates every edge from S to T and avoids every edge from T to S. Moreover, if we have a flow f and a cut (S,T) that satisfies this equality condition, f must be a maximum flow, and (S,T) must be a minimum cut.

Now the question is: from a Ford-Fulkerson's algorithm solution, how to find the associated Min-cut?

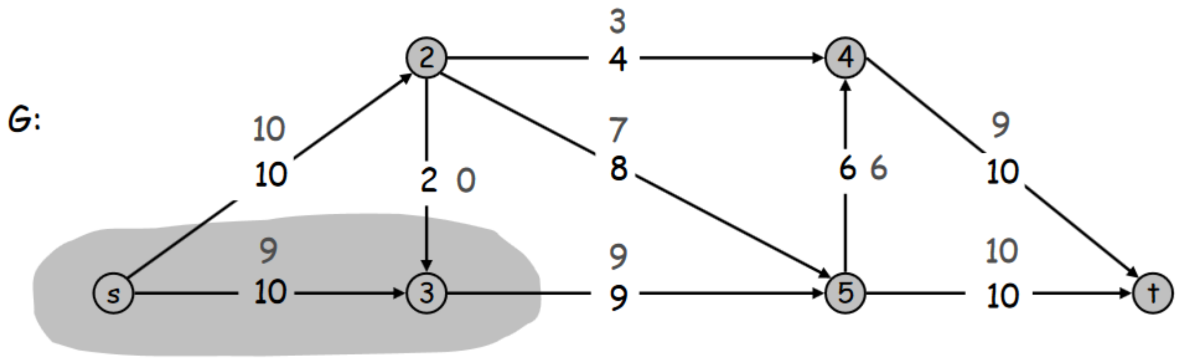




You can see the final step of the example shown in Practice 4.1. If you compute a search algorithm on the graph G_f, you will find a set of vertices.



On the graph G, this set is equal to:



If we apply the equation on this set, we found $f = 10 + 9 - 0 = 19$. Eureka! This cut is equal to the max-flow, we can conclude that we found a min-cut solution.

One of the magic tricks done by the min-cut is to find the bottleneck on a graph. Here, we can clearly see that the vertices s and 3 limit the total flow on the graph.