

# Practice 2.2: Transportation

## *Combinatorial optimization*

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### Modified Distribution Method

Consider the **transportation problem** presented in the following table.

		Distribution centre				
		D1	D2	D3	D4	Supply
Plant	P1	19	30	50	12	7
	P2	70	30	40	60	10
	P3	40	10	60	20	18
Requirement		5	8	7	15	

Determine the optimal solution of the above problem.

#### *Solution.*

An initial basic feasible solution is obtained by Matrix Minimum Method and is shown in below.

		Distribution centre				
		D1	D2	D3	D4	Supply
Plant	P1	19	30	50	12 <sup>7</sup>	7
	P2	70 <sup>3</sup>	30	40 <sup>7</sup>	60	10
	P3	40 <sup>2</sup>	10 <sup>8</sup>	60	20 <sup>8</sup>	18
Requirement		5	8	7	15	

#### *Initial basic feasible solution*

$$12 \times 7 + 70 \times 3 + 40 \times 7 + 40 \times 2 + 10 \times 8 + 20 \times 8 = \text{Rs. } 894.$$

Calculating  $u_i$  and  $v_j$  using  $u_i + v_j = c_{ij}$

Substituting  $u_1 = 0$ , we get

$$u_1 + v_4 = c_{14} \Rightarrow 0 + v_4 = 12 \text{ or } v_4 = 12$$

$$u_3 + v_4 = c_{34} \Rightarrow u_3 + 12 = 20 \text{ or } u_3 = 8$$

$$u_3 + v_2 = c_{32} \Rightarrow 8 + v_2 = 10 \text{ or } v_2 = 2$$

$$u_3 + v_1 = c_{31} \Rightarrow 8 + v_1 = 40 \text{ or } v_1 = 32$$

$$u_2 + v_1 = c_{21} \Rightarrow u_2 + 32 = 70 \text{ or } u_2 = 38$$

Distribution centre							
		D1	D2	D3	D4	Supply	$u_i$
Plant	P1	19	30	50	12 <sup>7</sup>	7	0
	P2	70 <sup>3</sup>	30	40 <sup>7</sup>	60	10	38
	P3	40 <sup>2</sup>	10 <sup>8</sup>	60	20 <sup>8</sup>	18	8
Requirement		5	8	7	15		
$v_j$		32	2	2	12		

Calculating **opportunity cost** using  $c_{ij} - (u_i + v_j)$

Unoccupied cells	Opportunity cost
$(P_1, D_1)$	$c_{11} - (u_1 + v_1) = 19 - (0 + 32) = -13$
$(P_1, D_2)$	$c_{12} - (u_1 + v_2) = 30 - (0 + 2) = 28$
$(P_1, D_3)$	$c_{13} - (u_1 + v_3) = 50 - (0 + 2) = 48$
$(P_2, D_2)$	$c_{22} - (u_2 + v_2) = 30 - (38 + 2) = -10$
$(P_2, D_4)$	$c_{14} - (u_2 + v_4) = 60 - (38 + 12) = 10$
$(P_3, D_3)$	$c_{33} - (u_3 + v_3) = 60 - (8 + 2) = 50$

		Distribution centre					
		D1	D2	D3	D4	Supply	$u_i$
Plant	P1	-13   19	28   30	48   50	12   7	7	0
	P2	70   3	-10   30	40   7	10   60	10	38
	P3	40   2	10   8	50   60	20   8	18	8
Requirement		5	8	7	15		
$v_j$		32	2	2	12		

Now choose the smallest (most) negative value from opportunity cost (i.e., -13) and draw a closed path from P1D1. The following table shows the closed path.

		Distribution centre					
		D1	D2	D3	D4	Supply	$u_i$
Plant	P1	-13   19 +	28   30	48   50	12   7 -	7	0
	P2	70   3	-10   30	40   7	10   60	10	38
	P3	40   2 -	10   8	50   60	20   8 +	18	8
Requirement		5	8	7	15		
$v_j$		32	2	2	12		

Choose the smallest value with a negative position on the closed path (i.e., 2), it indicates the number of units that can be shipped to the entering cell. Now add this quantity to all the cells on the corner points of the closed path marked with plus signs and subtract it from those cells marked with minus signs. In this way, an unoccupied cell becomes an occupied cell.

Now again calculate the values for  $u_i$  &  $v_j$  and opportunity cost. The resulting matrix is shown below.

		Distribution centre					
		D1	D2	D3	D4	Supply	$u_i$
Plant	P1	19   2	28   30	61   50	12   5	7	0
	P2	70   3	-23   30	40   7	-3   60	10	51
	P3	13   40	10   8	63   60	20   10	18	8
Requirement		5	8	7	15		
$v_j$		19	2	-11	12		

Choose the smallest (most) negative value from opportunity cost (i.e., -23). Now draw a closed path from P2D2 .

Distribution centre							
		D1	D2	D3	D4	Supply	$u_i$
Plant	P1	$\begin{matrix} + \\ 19 \end{matrix} \begin{matrix} 2 \\ \end{matrix}$	$\begin{matrix} 28 \\ \end{matrix} \begin{matrix}   \\ 30 \end{matrix}$	$\begin{matrix} 61 \\ \end{matrix} \begin{matrix}   \\ 50 \end{matrix}$	$\begin{matrix} - \\ 12 \end{matrix} \begin{matrix} 5 \\ \end{matrix}$	7	0
	P2	$\begin{matrix} - \\ 70 \end{matrix} \begin{matrix} 3 \\ \end{matrix}$	$\begin{matrix} -23 \\ \end{matrix} \begin{matrix}   \\ 30 \end{matrix} \begin{matrix} + \\ \end{matrix}$	$\begin{matrix} 40 \\ \end{matrix} \begin{matrix} 7 \\ \end{matrix}$	$\begin{matrix} -3 \\ \end{matrix} \begin{matrix}   \\ 60 \end{matrix}$	10	51
	P3	$\begin{matrix} 13 \\ \end{matrix} \begin{matrix}   \\ 40 \end{matrix}$	$\begin{matrix} 10 \\ \end{matrix} \begin{matrix} 8 \\ \end{matrix}$	$\begin{matrix} 63 \\ \end{matrix} \begin{matrix}   \\ 60 \end{matrix}$	$\begin{matrix} + \\ 20 \end{matrix} \begin{matrix} 10 \\ \end{matrix}$	18	8
Requirement		5	8	7	15		
$v_j$		19	2	-11	12		

Now again calculate the values for  $u_i$  &  $v_j$  and opportunity cost

Distribution centre							
		D1	D2	D3	D4	Supply	$u_i$
Plant	P1	$\begin{matrix} 19 \\ \end{matrix} \begin{matrix} 5 \\ \end{matrix}$	$\begin{matrix} 28 \\ \end{matrix} \begin{matrix}   \\ 30 \end{matrix}$	$\begin{matrix} 38 \\ \end{matrix} \begin{matrix}   \\ 50 \end{matrix}$	$\begin{matrix} 12 \\ \end{matrix} \begin{matrix} 2 \\ \end{matrix}$	7	0
	P2	$\begin{matrix} 23 \\ \end{matrix} \begin{matrix}   \\ 70 \end{matrix}$	$\begin{matrix} 30 \\ \end{matrix} \begin{matrix} 3 \\ \end{matrix}$	$\begin{matrix} 40 \\ \end{matrix} \begin{matrix} 7 \\ \end{matrix}$	$\begin{matrix} 20 \\ \end{matrix} \begin{matrix}   \\ 60 \end{matrix}$	10	28
	P3	$\begin{matrix} 13 \\ \end{matrix} \begin{matrix}   \\ 40 \end{matrix}$	$\begin{matrix} 10 \\ \end{matrix} \begin{matrix} 5 \\ \end{matrix}$	$\begin{matrix} 40 \\ \end{matrix} \begin{matrix}   \\ 60 \end{matrix}$	$\begin{matrix} 20 \\ \end{matrix} \begin{matrix} 13 \\ \end{matrix}$	18	8
Requirement		5	8	7	15		
$v_j$		19	2	12	12		

Since all the current opportunity costs are non-negative, this is the optimal solution. The minimum transportation cost is:  $19 \times 5 + 12 \times 2 + 30 \times 3 + 40 \times 7 + 10 \times 5 + 20 \times 13 = \text{Rs. } 799$