

Tutorial 1: Simplex method

Combinatorial Optimization

G.Guérard

Primal-Dual

Exercise 1

The advertising alternative for a company include television, radio, and newspaper advertisements. The costs and estimates for audience coverage are given in the table below.

	Television	Radio	Newspaper
Cost per advertisement	£2000	£300	£600
Audience per advertisement	100000	18000	40000

The local newspaper limits the number of weekly advertisements from a single company to ten. Moreover, in order to balance the advertising among the three types of media, no more than half of the total number of advertisements should occur on the radio, and at least 10% should occur on television. The weekly advertising budget is £18'200. How many advertisements should be run in each of the three types of media to maximize the total audience?

Exercise 2

A small petroleum company owns two refineries. Refinery 1 costs £20'000 per day to operate, and it can produce 400 barrels of high-grade oil, 300 barrels of medium-grade oil, and 200 barrels of low-grade oil each day. Refinery 2 costs £25'000 per day to operate, and it can produce 300, 400, 500 barrels (same order) each day. The company has orders totaling 25'000 barrels of high-grade oil, 27'000 barrels of medium-grade oil, and 30'000 barrels of low-grade oil. How many days should it run each refinery to minimize its costs and still refine enough oil to meet its orders?

Exercise 3

Consider the following LP problem:

$$\begin{aligned} \min \quad & -5x_1 - 4x_2 - 3x_3 \\ \text{s.t.} \quad & 2x_1 + 3x_2 + 1x_3 \geq 5 \\ & 4x_1 + 1x_2 + 2x_3 \leq 13 \\ & 3x_1 + 4x_2 + 2x_3 = 9 \\ & x_1, x_2, x_3 \geq 0 \end{aligned}$$

Use the CS conditions to verify whether or not the solution $\{1,1,1\}$ is a, optimal solution to the LP problem.

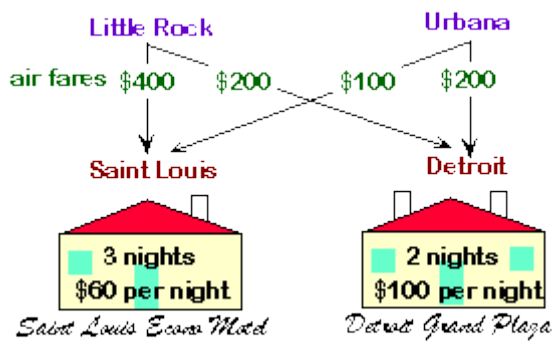
Exercise 4

Find the optimal value z of the LP problem below by inspecting its dual. Then, find the primal optimal solution using the CS conditions.

$$\begin{aligned} \min z &= 10x_1 + 4x_2 + 5x_3 \\ \text{s.t.} \quad &5x_1 - 7x_2 + 3x_3 \geq 50 \\ &x_1, x_2, x_3 \geq 0 \end{aligned}$$

Exercise 5

Your software company has launched the latest version of its web browser, "Java Cruise 4.0." As sales manager, you are planning to promote Java Cruise 4.0 by sending sales forces to software conventions running concurrently in Saint Louis and Detroit. You have 6 representatives available at each of your Little Rock, Ark. and Urbana, Ill. branches, and you would like to send at least 5 to the Saint Louis convention and at least 4 to the Detroit convention. The Saint Louis convention will last for three days, while the Detroit convention will last for two days. Air fares (per person) and hotel accommodation costs (per person) are shown in the following figure.



How many representatives should you send from each branch to each convention in order to minimize the total (air travel and accommodation) cost? What will the total cost amount to?

Shadow costs

Exercise 6

Consider the following Linear Programming model:

