

# Tutorial 3: Assignment

## *Combinatorial Optimization*

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### Exercise 1

The Atlantic Coast Grid deserves four cities. The office wants to assign four plants. The price to send energy from a plant to each city is described below:

	Raleigh	Atlanta	Durham	Clemson
Plant A	210	90	180	160
Plant B	100	70	130	200
Plant C	175	105	140	170
Plant D	80	65	105	120

A plant can deserve only one city, and a city can take energies only from one plant. Find the best assignment at lowest cost.

### Exercise 2

Permute the columns of a square matrix so as to minimize the sum of elements on the main diagonal.

8	16	15	91	64
83	42	93	75	27
76	95	75	81	50
20	42	96	90	24
38	28	2	15	81

### Exercise 3

Three robots {a,b,c} need to finish three tasks {t1, t2, t3} in the following grid. It take one day for a robot to move from one cell to one of its neighbors.

$t_3$				$t_1$
			<b>b</b>	
<b>c</b>				
			$t_2$	
<b>a</b>				

In the following table, we list the days that each robot can finish each task alone. The tasks need to be finished as soon as possible.

	$t_1$	$t_2$	$t_3$
a	10	20	15
b	30	30	20
c	15	10	10

### Exercise 4

Braneast Airlines must staff the daily flights between New York and Chicago shown in the table below. Each of Braneast's crews lives in either New York or Chicago. Each day a crew must fly one NY-Chicago and one Chicago-NY flight with at least one hour of downtime between flights.

Braneast wants to schedule the crews to minimize the total downtime. Set up an assignment problem that can be used to accomplish this goal. Of course, some assignments are not possible. Find the flight assignments that minimize the total downtime. How many crews should be based in each city? Assume that at the end of the day, each crew must be in his home city.

Flight	Leave Chicago	Arrive New York	Flight	Leave New York	Arrive Chicago
1	6 A.M.	10 A.M.	1	7 A.M.	9 A.M.
2	9 A.M.	1 P.M.	2	8 A.M.	10 A.M.
3	12 noon	4 P.M.	3	10 A.M.	12 noon
4	3 P.M.	7 P.M.	4	12 noon	2 P.M.
5	5 P.M.	9 P.M.	5	2 P.M.	4 P.M.
6	7 P.M.	11 P.M.	6	4 P.M.	6 P.M.
7	8 P.M.	12 midnight	7	6 P.M.	8 P.M.

### Exercise 5

Consider the data of table below. If a crew based in Mumbai arrives at Delhi on a given flight, it must return to Mumbai on a later flight. Assume that for any given pairing, the crew will be based in the city that results in the smaller layover. The problem is to find the pairings so as to minimize the time on ground away from home, subject to a minimum interval of one hour between arrival and departure. Given the pairs of flights, where should the crews be based?

<i>Flight No.</i>	<i>From</i>	<i>Time of departure</i>	<i>To</i>	<i>Arrival time</i>
1	Mumbai	6.00 am	Delhi	8.00 am
2	Mumbai	7.00 am	Delhi	9.00 am
3	Mumbai	10.00 am	Delhi	12.00 noon
4	Mumbai	2.00 pm	Delhi	4.00 pm
5	Mumbai	6.00 pm	Delhi	8.00 pm
6	Mumbai	8.00 pm	Delhi	10.00 pm
7	Delhi	6.00 am	Mumbai	8.00 am
8	Delhi	8.00 am	Mumbai	10.00 pm
9	Delhi	11.00 am	Mumbai	1.00 pm
10	Delhi	3.00 pm	Mumbai	5.00 pm
11	Delhi	6.00 pm	Mumbai	8.00 pm
12	Delhi	9.00 pm	Mumbai	11.00 pm

### Exercise 6

Resolve the following problem as an assignment problem.

$$\text{Minimize } 4X_{11}+6X_{12}+5X_{13}+5X_{14}+7X_{21}+4X_{22}+5X_{23}+6X_{24} +4X_{31}+7X_{32}+6X_{33}+4X_{34} +5X_{41}+3X_{42}+4X_{43}+7X_{44}$$

$$\text{St. } X_{11}+X_{12}+X_{13}+X_{14}=1$$

$$X_{21}+X_{22}+X_{23}+X_{24}=1$$

$$X_{31}+X_{32}+X_{33}+X_{34}=1$$

$$X_{41}+X_{42}+X_{43}+X_{44}=1$$

$$X_{11}+X_{21}+X_{31}+X_{41}=1$$

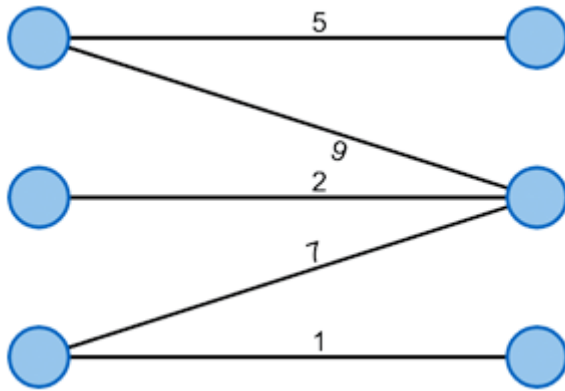
$$X_{12}+X_{22}+X_{32}+X_{42}=1$$

$$X_{13}+X_{23}+X_{33}+X_{43}=1$$

$$X_{14}+X_{24}+X_{34}+X_{44}=1$$

### Exercise 7

Resolve the following problem as an assignment problem.



### Exercise 8

The Art History Department wishes to offer six courses in a semester. There are seven professors in the department, each of which can teach only certain courses, as shown in the table. Is it possible to assign the six courses to the professors so that no professor teaches more than one course?

Course	Professor
Antique	Ant, Bat, Cat, Dodo
Renaissance	Ant, Frog, Gnat
Baroque	Ant, Frog
Impressionism	Frog, Gnat
Modern	Cat, Gnat, Hog
Contemporary	Ant, Gnat