Tutorial 2: Transportation
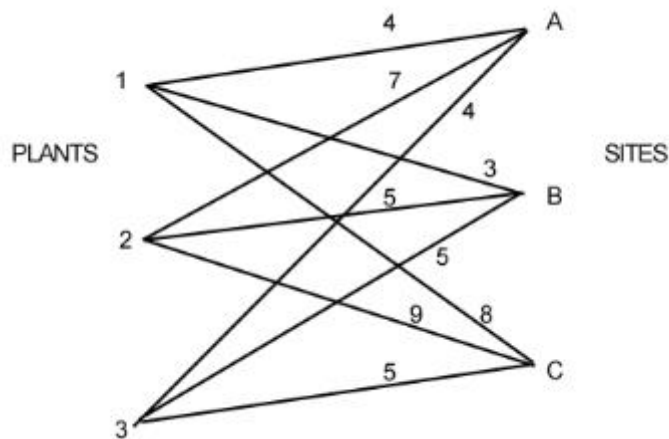## *Combinatorial Optimization*

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> Vous pouvez commencer le TP par de simples questions de cours ou des exemples « sans réflexion ».

Exercise 1

A company has to transport supplies from plants to construction site. The three plants have 300, 300, 100 supplies capacity respectively. And the three construction sites demand 200, 200, 300 respectively.

The transporting costs are shown in the following graph:

Find how to allocate the supplies.

Solution

	Sites			
From \ To	A	B	C	Supply (Availability)
Plants 1	4	3	8	300
2	7	5	9	300
3	4	5	5	100
Demand (requirement)	200	200	300	

Exercise 2

Wheat is harvested in the Midwest and stored in grain elevators in three different cities—Kansas City, Omaha, and Des Moines. These grain elevators supply three flour mills, located in Chicago, St.

Louis, and Cincinnati. Grain is shipped to the mills in railroad cars, each car capable of holding one ton of wheat. Each grain elevator is able to supply the following number of tons (i.e., railroad cars) of wheat to the mills on a monthly basis.

Grain Elevator	Supply
1. Kansas City	150
2. Omaha	175
3. Des Moines	275

Each mill demands the following number of tons of wheat per month.

Mill	Demand
A. Chicago	200
B. St. Louis	100
C. Cincinnati	300

The cost of transporting one ton of wheat from each grain elevator (source) to each mill (destination) differs according to the distance and rail system. These costs are shown in the following table. For example, the cost of shipping one ton of wheat from the grain elevator at Omaha to the mill at Chicago is $7.

	Mill		
Grain Elevator	A. Chicago	B. St. Louis	C. Cincinnati
1. Kansas City	$6	$ 8	$10
2. Omaha	7	11	11
3. Des Moines	4	5	12

The problem is to determine how many tons of wheat to transport from each grain elevator to each mill on a monthly basis in order to minimize the total cost of transportation. Is there any alternative solution? If yes, which one?

Solution

From \ To	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Demand	200	100	300	600

To From	A	B	C	Supply
1	6	8	10 150	150
2	7 25	11	11 150	175
3	4 175	5 100	12	275
Demand	200	100	300	600

Exercise 3

From the linear program below, formulate the transportation problem and solve it.

$$\text{minimize } Z = \$6x_{1A} + 8x_{1B} + 10x_{1C} + 7x_{2A} + 11x_{2B} + 11x_{2C} + 4x_{3A} + 5x_{3B} + 12x_{3C}$$

subject to

$$x_{1A} + x_{1B} + x_{1C} = 150$$
$$x_{2A} + x_{2B} + x_{2C} = 175$$
$$x_{3A} + x_{3B} + x_{3C} = 275$$
$$x_{1A} + x_{2A} + x_{3A} = 200$$
$$x_{1B} + x_{2B} + x_{3B} = 100$$
$$x_{1C} + x_{2C} + x_{3C} = 300$$
$$x_{ij} \geq 0$$

Is there any alternative solution? If yes, which one?

Solution
Same as exercise 2.

Exercise 4

Take the same problem as exercise 2. Change the demand at Cincinnati to 350 tons. Solve it.

Solution

To From	A	B	C	Supply
1	6	8	10	150
2	7	11	11	175
3	4	5	12	275
Dummy	0	0	0	50
Demand	200	100	350	650

Exercise 5

Resolve the following LP problem:

$$\text{Minimize} \quad 3x_{11} + 2x_{12} + x_{21} + 5x_{22} + 2x_{31} + 4x_{32}$$

Subject to:

$$
\begin{array}{llllll}
x_{11} & +x_{12} & & & & \leq 45 \quad (1) \\
& & x_{21} & +x_{22} & & \leq 60 \quad (2) \\
& & & & x_{31} +x_{32} & \leq 35 \quad (3) \\
x_{11} & & +x_{21} & & +x_{31} & \geq 50 \quad (4) \\
& x_{12} & & +x_{22} & +x_{32} & \geq 60 \quad (5)
\end{array}
$$

$$x_{ij} \geq 0 \quad \text{for } i = 1, 2, 3 \text{ and } j = 1, 2.$$

Solution

We introduce a dummy for j because supply(45+60+35)-demand(50+60)=30:

$$\text{Minimize} \quad 3x_{11} + 2x_{12} \qquad +x_{21} + 5x_{22} \qquad +2x_{31} + 4x_{32}$$

Subject to:

$$
\begin{array}{lllllll}
x_{11} & +x_{12} & +x_{13} & & & & = 45 \\
& & & x_{21} +x_{22} +x_{23} & & & = 60 \\
& & & & x_{31} +x_{32} +x_{33} & & = 35 \\
x_{11} & & & +x_{21} & & +x_{31} & = 50 \\
& x_{12} & & & +x_{22} & +x_{32} & = 60 \\
& & x_{13} & & +x_{23} & +x_{33} & = 30
\end{array}
$$

$$x_{ij} \geq 0 \quad \text{for } i = 1, 2, 3 \text{ and } j = 1, 2, 3.$$

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Sinks

	1	2	3	Supply
1	3	2 — 45	0	45
Sources 2	1 — 50	5	0 — 10	60
3	5	4 — 15	0 — 20	35
Demand	50	60	30	

Exercise 6

The Atlantic Coast Conference has four basketball games on a particular night. The conference office wants to assign four teams of officials to the four games in a way that will minimize the total distance traveled by the officials. The distances in miles for each team of officials to each game location are shown in Table below:

Officials	Game Sites			
	RALEIGH	ATLANTA	DURHAM	CLEMSON
A	210	90	180	160
B	100	70	130	200
C	175	105	140	170
D	80	65	105	120

Solve this problem as a transportation problem.

Solution

Each supply and each demand are equal to 1. Solve it as a transportation problem.